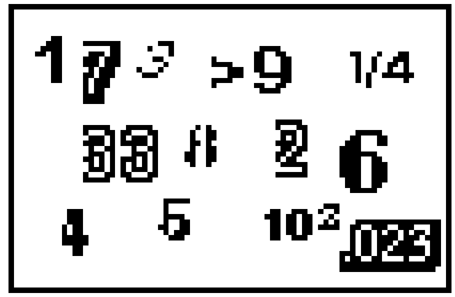
Warm-Up 3

The Numbers Game



Duration	1 class period
Grade Level	10-12
Key Terms/ Concepts	Concentration Contaminants Toxic Unit of Measure
Suggested Subjects	Mathematics

Purpose

Students gain an appreciation for the part-per-million and part-per-billion units used to measure contaminant concentrations in the environment. Students learn to calculate these ratios and analyze a sample chemical spill to determine if cleanup action is necessary.

Background

Some toxic substances are dangerous even in very small amounts. "Part-per-million" (ppm) and "part-per-billion" (ppb) are the units of measure scientists use to describe the concentration of a hazardous substance or contaminant found within a large volume of another substance. For instance, your could find 500 ppm of a pesticide in a lake.

Expressing the amount of contamination in ppm or ppb is measuring the concentration of the substance. This way, a scientist can take a relatively small sample of water, from the lake in our example, and measure the concentration of one or more contaminants in that sample, then assume that the concentration is the same in the whole lake without testing the entire lake.

For more information on the science of detecting and measuring contamination, see the Suggested Reading list found at the end of the Haz-Ed materials. Other Haz-Ed materials that are related to this topic include *Activity 7: Identifying Risks at a Superfund Site* and *Activity 9: Making Decisions About Hazardous Waste Cleanup.*

Preparation

1. Gather the following materials:

   - Copies for each student of the Student Worksheet, *The Numbers Game. (An answer sheet for your use is included at the end of this lesson.)*

Procedure

1. Hand out the Student Worksheet, *The Numbers Game*, and have the students take the quiz in Part A. Part A is intended to gauge the students' intuitive grasp of how small a "part per million" and a "part per billion" are. Instruct the students to guess if necessary to answer these three questions. They will actually calculate the correct answers in Part B.

2. After they have completed the quiz, go on to Part B. Work with them to calculate each answer choice and, from that information, determine the correct answers to the quiz in Part A.

3. Finally, work through the Lake Jasmine spill scenario in Part C with the students.

Instructor's Answer Key

Warm-Up 3: The Numbers Game

Correct answers are boxed.

Part A

Just how small is a part per million? A part per billion? Answer the following three questions based on your "gut reaction." Guess if you need to.

1. One part per million is equivalent to **1 minute** in
   a. 1 day $\boxed{\text{b. 2 years}}$ c. 6 weeks

2. One part per billion is equivalent to **1 second** in
   a. 3 weeks b. 17 months $\boxed{\text{c. 32 years}}$

Part B

Now go back and calculate each of the answers you chose in Part A. Use the procedure below for each calculation.

To calculate the relationship between 2 quantities, first **convert both quantities to the same unit of measure**. For example, to compare years to seconds, convert the years to seconds. To do this, convert the years to days, then the days to hours, the hours to minutes and the minutes to seconds:

$$\frac{\text{365 days}}{\text{1 year}} \times \frac{\text{24 hours}}{\text{1 day}} = \frac{\text{8,760 hours}}{\text{1 year}}$$

$$\frac{\text{8,760 hours}}{\text{1 year}} \times \frac{\text{60 minutes}}{\text{1 hour}} = \frac{\text{525,600 minutes}}{\text{1 year}}$$

$$\frac{\text{525,600 minutes}}{\text{1 year}} \times \frac{\text{60 seconds}}{\text{1 minute}} = \frac{\text{31,536,000 seconds}}{\text{1 year}}$$

1. Use the space below to calculate (a) 1 minute per day, (b) 1 minute per 2 years, and (c) 1 minute per 6 weeks to find the answer to question 1 of Part A. After you have completed the conversion to the same units (e.g., expressing hours, days, or weeks in minutes or seconds), you may have to round your answers to the nearest thousand, million, or billion.

a) $$\frac{\text{1 minute}}{\text{1 day}} \times \frac{\text{1 day}}{\text{24 hours}} \times \frac{\text{1 hour}}{\text{60 minutes}} = \frac{\text{1 minute}}{\text{1,440 minutes}} \approx \frac{1}{1{,}500} \Rightarrow \text{1 part per 1,500}$$

b) $$\boxed{\frac{\text{1 minute}}{\text{2 years}} \times \frac{\text{1 year}}{\text{365 days}} \times \frac{\text{1 day}}{\text{24 hours}} \times \frac{\text{1 hour}}{\text{60 minutes}} = \frac{\text{1 minute}}{\text{1,051,200 minutes}} \approx \frac{1}{1{,}000{,}000} \Rightarrow \text{1 part per million}}$$

c) $$\frac{\text{1 minute}}{\text{6 weeks}} \times \frac{\text{1 week}}{\text{7 days}} \times \frac{\text{1 day}}{\text{24 hours}} \times \frac{\text{1 hour}}{\text{60 minutes}} = \frac{\text{1 minute}}{\text{60,480 minutes}} \approx \frac{1}{60{,}000} \Rightarrow \text{1 part per 60,000}$$

2. Use the space below to calculate (a) 1 second per 3 weeks, (b) 1 second per 17 years, and (c) 1 second per 32 years to find the answer to question 2 of Part A.

1) $$\frac{\text{1 second}}{\text{3 weeks}} \times \frac{\text{1 week}}{\text{7 days}} \times \frac{\text{1 day}}{\text{24 hours}} = \frac{\text{1 hour}}{\text{60 minutes}} \times \frac{\text{1 minute}}{\text{60 seconds}} = \frac{\text{1 second}}{\text{18,144,400 seconds}}$$

$$\approx \frac{1}{18{,}000{,}000} \Rightarrow \text{1 part per 18 million}$$

2) $$\frac{\text{1 second}}{\text{17 years}} \times \frac{\text{1 year}}{\text{365 days}} \times \frac{\text{1 day}}{\text{24 hours}} = \frac{\text{1 hour}}{\text{3,600 seconds}} = \frac{\text{1 second}}{\text{536,112,000 seconds}}$$

$$\approx \frac{1}{500{,}000{,}000} \Rightarrow \text{1 part per 500,000,000}$$

3) $$\boxed{\frac{\text{1 second}}{\text{32 years}} \times \frac{\text{1 year}}{\text{365 days}} \times \frac{\text{1 day}}{\text{24 hours}} = \frac{\text{1 hour}}{\text{3,600 seconds}} = \frac{\text{1 second}}{\text{1,009,152,000 seconds}}}$$

$$\boxed{\approx \frac{1}{1{,}000{,}000{,}000} \Rightarrow \text{1 part per billion}}$$

Part C

If the conversion of units leads to a fraction with a numerator other than 1, a different method can be used to determine parts per million or parts per billion. Be sure your fraction has a smaller number on top and larger number on the bottom and divide.

To express the decimal answer in parts per million, move the decimal point 6 places to the right. To express the answer in parts per billion, move the decimal point 9 places to the right.

Example 1: $\frac{\text{20 ounces}}{\text{100 pounds}} = \frac{\text{2 ounces}}{\text{(100 x 16 ounces)}} = \frac{\text{2 ounces}}{\text{1,600 ounces}} \approx \frac{2}{1{,}600}$ => 0.00125

Moving the decimal place 6 places to the right gives 1,250 parts per million.
Moving the decimal place 9 places to the right gives 1,250,000 parts per billion. (You would probably not see a number this large expressed in parts per billion. It is better expressed as a smaller number of parts per million.)

Example 2: $\frac{\text{11 ounces}}{\text{10 tons}} = \frac{\text{11 ounces}}{\text{20,000 pounds}} = \frac{\text{11 ounces}}{\text{(20,000 x 16) ounces}} = \frac{\text{11 ounces}}{\text{320,000 ounces}}$ => 0.00003437

Moving the decimal place 6 places to the right gives 34.37, or about 34.4 parts per million. Moving the decimal place 9 places to the right gives 34,370, or about 34,000 parts per billion.

(NOTE: Depending on the skill level of your class, you may need to let students practice calculating and converting measures of volume. Since students may associate volume with rectangular objects, you may want to use a swimming pool for this sample problem. Assume that a 50-gallon container of chlorine is spilled into a swimming pool, which is 100 feet long by 50 feet wide by 10 feet deep.)

Based on the scenario described below and the table of legally allowable concentrations of contaminants in surface water, decide whether local public health officials should take measures to keep vacationers near Lake Jasmine out of the water.

Allowable Quantities:	Fuel Oil A	2.2 ppm in recreational waters
(concentrations of contaminants	Pesticide B	4.7 ppm in recreational waters
above these levels require action)	Solvent C	1.3 ppm in recreational waters

Conversion Table: 1 acre = 43,560 square feet
1 gallon = 0.1337 cubic feet
1 cubic foot = 7.48 gallons

SCENARIO

Lake Jasmine is a 20-acre lake with an average depth of 30 feet. Yesterday afternoon, four 55-gallon drums of Fuel Oil A and six 55-gallon drums of Solvent C fell off a truck during an accident, rolled into Lake Jasmine and burst open on the rocky shore. The entire contents of all the drums spilled into the lake.

STEP 1

Calculate the concentration of each contaminant (in ppm) in Lake Jasmine. To do this you must compare the volume of the contaminants (gallons) to the volume of the lake (cubic feet).

Start by converting both to cubic feet. To calculate the volume of the lake, multiply the area (in square feet) by the depth (in feet) to get cubic feet.

Calculate volume of contaminants:

4 x 55 gallons Fuel Oil A x 1 cubic foot/7.48 gallons = 29.42 cubic feet Fuel Oil A
and
6 x 55 gallons Solvent C x 1 cubic foot/7.48 gallons = 44.12 cubic feet of Solvent C

Calculate volume of Lake Jasmine:

20 acres x 30 feet x 43,560 square feet/acre = 26,136,000 cubic feet of water in Lake Jasmine

Comparison:

29.42 cubic feet Fuel Oil A/26,136,000 cubic feet Lake Jasmine
= 29.42/26,136,000
= 0.0000011
Moving the decimal point 6 places to the right gives 1.1 ppm Fuel Oil A

44.12 cubic feet Solvent C/26,136,000 cubic feet Lake Jasmine
= 44.12/26,136,000
= 0.0000016
Moving the decimal point 6 places to the right gives 1.6 ppm Solvent C

STEP 2
Compare these levels to the values in the chart of allowable quantities to see if they exceed the legally allowable levels.

Allowable Quantities*: (concentrations of contaminants above these levels require action)*

Fuel Oil A	*2.2 ppm in recreational waters*
Pesticide B	*4.7 ppm in recreational waters*
Solvent C	*1.3 ppm in recreational waters*

1.1 ppm Fuel Oil A does not exceed the allowable concentration of 2.2 ppm. If that were the only chemical spilled, no action would be necessary.

However, since 1.6 ppm Solvent C does exceed the limit, local health officials will have to keep Lake Jasmine residents out of the water until the levels of contaminants are lowered.

The Numbers Game

Part A

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After you have completed the conversion to the same units (e.g., expressing hours, days, or weeks in minutes or seconds), you may have to round your answer to the nearest thousand, million, or billion.

Use the space below to calculate (a) 1 second per 3 weeks, (b) 1 second per 17 years, and (c) 1 second per 32 years to find the answer to question 2 of Part A.

Part C

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